S.E. (Comp & IT) - IV (CBSGS)

A.M. IV

15/5/2015

Q.P. Code : 3541

(3 Hours) **[Total Marks : 80**

N.B.: (1) Question **No.1** is **compulsory**.
(2) Attempt any **three** questions from Question No. **2** to **6**.
(3) Use of stastical Tables permitted.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) Show that $\int_C \log z \, dz = 2\pi i$, where C is the unit circle in the z - plane. **5**

(b) If $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ then find the eigen values of $4A^{-1} + 3A + 2I$. **5**

(c) It is given that the means of x and y are 5 and 10. If the line of regression of y on x is parallel to the line $20y = 9x + 40$, estimate the value of y for $x = 30$. **5**

(d) Find the dual of the following L.P.P. **5**

Maximise $Z = 2x_1 - x_2 + 3x_3$

Subject to $x_1 - 2x_2 + x_3 \geq 4$

$2x_1 + x_3 \leq 10$

$x_1 + x_2 + 3x_3 = 20$

$x_1, x_3 \leq 0$, x_2 unrestricted.

2. (a) Evalute $\int_C \frac{z+2}{z^3 - 2z^2} dz$, where C is the circle $|Z - 2 - i| = 2$ **6**

(b) Show that $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory. **6**

(c) In a distribution exactly normal 7% of items are under 35 and 89% of the items are under 63. Find the probability that an item selected at random lies between 45 & 56. **8**

3. (a) A continuous random variable has probability density function $f(x) = 6(x - x)^2$, $0 \leq x \leq$ i. Find (i) mean (ii) variance. **6**

(b) Solve the following L.P.P. by simplex method **6**

Maximise $Z = 4x_1 + 3x_2 + 6x_3$

Subject to $2x_1 + 3x_2 + 2x_3 \leq 440$

$4x_1 + 3x_3 \leq 470$

$2x_1 + 5x_2 \leq 430$

$x_1, x_2, x_3 \leq 0$

3. (c) Find all possible Laurent's expansions of the function 8

$$f(z)=\frac{7z-2}{z(z-2)(z+1)} \quad \text{about} \quad z=-1$$

4. (a) Find the moment generating function of Binomial distribution & hence find mean and variance. 6

(b) Calculate the correlation coefficient from the following data : 6

x :	100	200	300	400	500
y :	30	40	50	60	70

(c) Show that the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalisable. Find the transforming matrix and the diagonal matrix. 8

5. (a) Ten individuals are chosen at random from a population and their heights are found to be 63, 63, 64, 65, 66, 69,69, 70, 70, 71 inches. Discuss the suggestion that the mean height of the universe is 65 inches. 6

(b) Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)^3}$, $a > 0$ using contour integration. 6

(c) Use Kuhn - Tucker conditions to solve the following N.L.P.P. 8

Maximise $Z = 8x_1 + 10x_2 - x_1^2 - x_2^2$

subject to $3x_1 + 2x_2 \leq 6$

$x_1, x_2 \geq 0$

6. (a) A die was thrown 132 times and the following frequencies were observed. 6

No. obtained :	1	2	3	4	5	6	Total
Frequency :	15	20	25	15	29	28	132

(b) Using duality solve the following L. P. P. 6

Maximise $Z = 5x_1 - 2x_2 + 3x_3$

Subject to $2x_1 + 2x_2 - x_3 \geq 2$

$3x_1 - 4x_2 \leq 3$

$x_1 + 3x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

(c) (i) A random sample of 50 items gives the mean 6.2 and standard deviation 10.24, can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance ? 4

(ii) Find the M.G.F. of the following distribution. 4

X :	-2	3	1
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

Hence find first four central moments.
